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INTEGRATION OF FIVE DIFFERENTIAL FORMS.

BY ARTEMAS MARTIN, M. A., ERIE, PA.

THE following forms, which sometimes occur in the solution of problems, are not integrated in any of the text-books on the Calculus to which I have access.

1. To integrate $dI_1 = \dfrac{xdx}{(m^2 - x^2)(n^2 - x^2)^{\frac{1}{2}}}$

Let $m^2 - x^2 = y^2$, then $xdx = -ydy$, $n^2 - x^2 = y^2 - m^2 + n^2$ and

$$I_1 = \int \frac{-dy}{y\sqrt{(y^2 - m^2 + n^2)}} = \frac{1}{\sqrt{(m^2 - n^2)}} \sin^{-1}\left(\frac{\sqrt{(m^2 - n^2)}}{y}\right),$$

$$= \frac{1}{\sqrt{(m^2 - n^2)}} \sin^{-1}\left(\frac{m^2 - n^2}{m^2 - x^2}\right)^{\frac{1}{2}}, \text{ when } m > n; \text{ and}$$

$$= \frac{1}{\sqrt{(n^2 - m^2)}} \log\left(\frac{\sqrt{(n^2 - m^2)} + \sqrt{(y^2 - m^2 + n^2)}}{y}\right)$$

$$= \frac{1}{\sqrt{(n^2 - m^2)}} \log\left(\frac{\sqrt{(n^2 - m^2)} + \sqrt{(n^2 - x^2)}}{\sqrt{(m^2 - x^2)}}\right), \text{ when } m < n.$$

2. To integrate $dI_2 = \dfrac{dx}{(m^2 - x^2)(n^2 - x^2)^{\frac{1}{2}}}$.

Let $n^2 - x^2 = x^2y^2$, then $x = \dfrac{n}{\sqrt{(1 + y^2)}}$, $y = \dfrac{\sqrt{(n^2 - x^2)}}{x}$, $dx = -\dfrac{ny\,dy}{(1 + y^2)^{\frac{3}{2}}}$,

$\sqrt{(n^2 - x^2)} = xy = \dfrac{ny}{\sqrt{(1 + y^2)}}$, $m^2 - x^2 = m^2 - \dfrac{n^2}{1 + y^2} = \dfrac{m^2 - n^2 + m^2y^2}{1 + y^2}$;

$$\therefore \quad I_2 = \int \frac{-ny\,dy}{(1 + y^2)^{\frac{3}{2}}} \times \frac{1 + y^2}{m^2 - n^2 + m^2y^2} \times \frac{\sqrt{(1 + y^2)}}{ny},$$

$$= \int \frac{-dy}{m^2y^2 + m^2 - n^2} = \frac{1}{m\sqrt{(m^2 - n^2)}} \tan^{-1}\left(\frac{\sqrt{(m^2 - n^2)}}{my}\right),$$

$$= \frac{1}{m\sqrt{(m^2 - n^2)}} \tan^{-1}\left(\frac{x\sqrt{(m^2 - n^2)}}{m\sqrt{(n^2 - x^2)}}\right),$$

$$= \frac{1}{m\sqrt{(m^2 - n^2)}} \sin^{-1}\left(\frac{x\sqrt{(m^2 - n^2)}}{n\sqrt{(m^2 - x^2)}}\right), \text{ when } m > n;$$

and

$$= \frac{1}{2m\sqrt{(n^2 - m^2)}} \log\left(\frac{my + \sqrt{(n^2 - m^2)}}{my - \sqrt{(n^2 - m^2)}}\right),$$

$$= \frac{1}{m\sqrt{(n^2 - m^2)}} \log\left(\frac{m\sqrt{(n^2 - x^2)} + x\sqrt{(n^2 - m^2)}}{n\sqrt{(m^2 - x^2)}}\right), \text{ when } m < n.$$

3. To integrate $dI_3 = \sin^{-1}\left(\frac{b^2}{a^2-x^2}\right)^{\frac{1}{2}}dx.$

By the method of parts,

$$I_3 = x\sin^{-1}\left(\frac{b^2}{a^2-x^2}\right)^{\frac{1}{2}} - \int\frac{bx^2dx}{(a^2-x^2)(a^2-b^2-x^2)^{\frac{1}{2}}}.$$

But $-\int\frac{bx^2dx}{(a-x^2)(a^2-b^2-x^2)^{\frac{1}{2}}} = \int\frac{bdx}{(a^2-b^2-x^2)^{\frac{1}{2}}} - \int\frac{a^2bdx}{(a^2-x^2)(a^2-b^2-x^2)^{\frac{1}{2}}};$

$$\therefore\ I_3 = x\sin^{-1}\left(\frac{b^2}{a^2-x^2}\right)^{\frac{1}{2}} + b\sin^{-1}\left(\frac{x^2}{a^2-b^2}\right)^{\frac{1}{2}} - a\sin^{-1}\left(\frac{b^2x^2}{(a^2-b^2)(a^2-x^2)}\right)^{\frac{1}{2}}$$

4. To integrate $dI_4 = \sin^{-1}\left(\frac{x^2}{a^2-x^2}\right)dx.$

$$\begin{aligned}I_4 &= x\sin^{-1}\left(\frac{x^2}{a^2-x_2}\right) - \int\frac{2ax^2dx}{(a^2-x^2)(a^2-2x^2)^{\frac{1}{2}}},\\ &= x\sin^{-1}\left(\frac{x^2}{a^2-x^2}\right) + a\sqrt{2}\int\frac{dx}{(\frac{1}{2}a^2-x^2)^{\frac{1}{2}}} - a^3\sqrt{2}\int\frac{dx}{(a^2-x^2)(\frac{1}{2}a^2-x^2)^{\frac{1}{2}}}\\ &= x\sin^{-1}\left(\frac{x^2}{a^2-x^2}\right) + a\sqrt{2}\sin^{-1}\left(\frac{x\sqrt{2}}{a}\right) - 2a\sin^{-1}\left(\frac{x^2}{a^2-x^2}\right)^{\frac{1}{2}}.\end{aligned}$$

5. To integrate $dI_5 = \sin^{-1}\left(\frac{x^2}{a^2-x^2}\right)^{\frac{1}{2}}dx.$

$$\begin{aligned}I_5 &= x\sin^{-1}\left(\frac{x^2}{a^2-x^2}\right)^{\frac{1}{2}} - a^2\int\frac{xdx}{(a^2-x^2)(a^2-2x^2)^{\frac{1}{2}}}),\\ &= x\sin^{-1}\left(\frac{x^2}{a^2-x^2}\right)^{\frac{1}{2}} - a\sqrt{2}\sin^{-1}\left(\frac{a^2}{2(a^2-x^2)}\right)^{\frac{1}{2}}.\end{aligned}$$

NOTE.—IN reference to the paragraph at the bottom of page 141 of the ANALYST for September, Prof. Abbe writes that when, in February, 1872, he wrote the article referred to, he desired only to bring to notice Dr. Adrain's early and original investigations and therefore selected for re-publication that one of the two proofs which its author seems to consider most satisfactory. The omission was not accidental — nor would it have been made had he duly considered the historical points that have been subsequently developed by Glaisher and Merriman.

QUERY. (BY REQUEST.) — In Salmon's Higher Plane Curves, 2nd ed. p. 180, is found the following proposition:—"A right angle has the side *GF* of fixed length, the point *F* moves along the fixed line *CI*, while the side *GH* passes through the fixed point *E*, a pencil at the middle point of *GF* will describe the Cissoid." How is this demonstrated?